Intro to Probability
Day 5
(More Difficult Probability Problems / Counting Techniques)

Summary of Formulas

$$P(E) = \frac{|E|}{|S|}$$

Classical Method: Use if all outcomes are equally likely and if you know |E| and |S|

$$P(E) = \frac{\#\ of\ times\ event\ occured}{\#\ of\ times\ experiment\ was\ performed}$$

Empirical Method: Use when trying to ESTIMATE a probability when you can't calculate it another way. Repeat the experiment many times and find the percentage of times the event E occured

---

$$P(A \cup B) = P(A) + P(B)$$

You can ONLY use this formula if the events A and B are DISJOINT

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Use this formula if the events A and B are NOT DISJOINT

$$P(A \cap B) = P(A) \cdot P(B)$$

You can ONLY use this formula if the events A and B are INDEPENDENT

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Use this formula if the events A and B are NOT INDEPENDENT

$$P(\bar{A}) = 1 - P(A)$$

$$P(at\ least\ one) = 1 - P(none)$$

More Complicated Probability Examples

REPHRASE THE QUESTION
using the words
AND, OR, or NOT

Ex 17 (book hw sec. 5.4 #27): **Playing a CD on the Random Setting**

Suppose that a compact disc (CD) you purchased has 13 tracks. After listening to the CD, you decide that you like 5 songs. With the random feature on your CD player, each of the 13 songs is played once in random order. Find the probability that among the first 2 songs played

a) You like both of them
b) You like neither of them
c) You like exactly one of them
d) Redo (a)-(c) if a song can be replayed before all 13 songs are played (if, for example, track 2 can be played twice in a row)

Ex 18:

Suppose you draw 2 cards from a standard poker deck. Find the probability that the total of the 2 cards is 20 if

a) The cards are drawn without replacement

b) The cards are drawn with replacement

Ex 19:

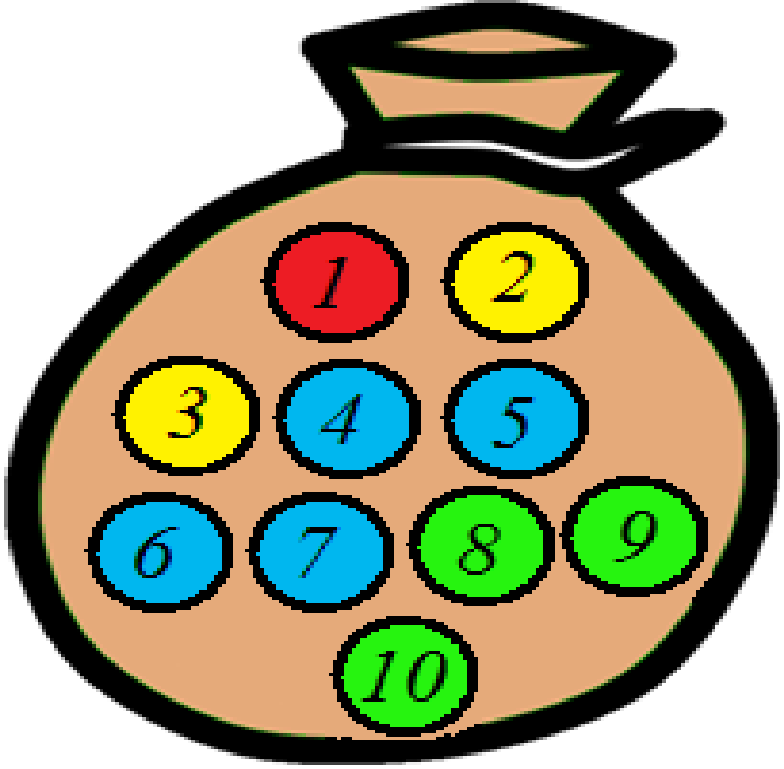
Suppose you draw 3 cards from a standard poker deck with replacement.

a) What is the probability that none are aces?
b) What is the probability that at least one are aces?
c) What is the probability that all are aces?
d) What is the probability that at least one is not an ace?
e) Repeat parts (a)-(d) for the situation where that cards are drawn without replacement

Ex 20:

Experiment

Draw 2 balls from the bag one by one without replacement



Question:

a) What is the probability that the 2nd ball drawn out of the bag is blue?

Ex 21 (book hw sec. 5.4 #33): **The Birthday Problem**

a) What is the probability that in a group of 30 people, everyone has a different birthday (ignoring leap years)

b) What is the probability that in a group of 30 people, at least 2 people have the same birthday (ignoring leap years)

Section 5.5: Counting Techniques

Multiplication Rule for Counting

Suppose an object can be built in 2 steps.
If there are

m ways to complete step 1

and

n ways to complete step 2,

then

total number of objects is $m \cdot n$

Note: This rule can be extended for objects that take 3 or more steps to build.

For examples 1-7, list a few outcomes of the experiment and count the total number of outcomes of the experiment

Ex 1:
Experiment
Flip a single coin four times

Ex 2:
Experiment
Flip a single coin once then roll a single die once

Ex 3:
Experiment
Roll a single die twice (or roll a pair of dice once)

Ex 4:

Experiment

Draw 3 cards from a deck one by one with replacement

Ex 5:

Experiment

Draw 3 cards from a deck one by one without replacement

Ex 6:

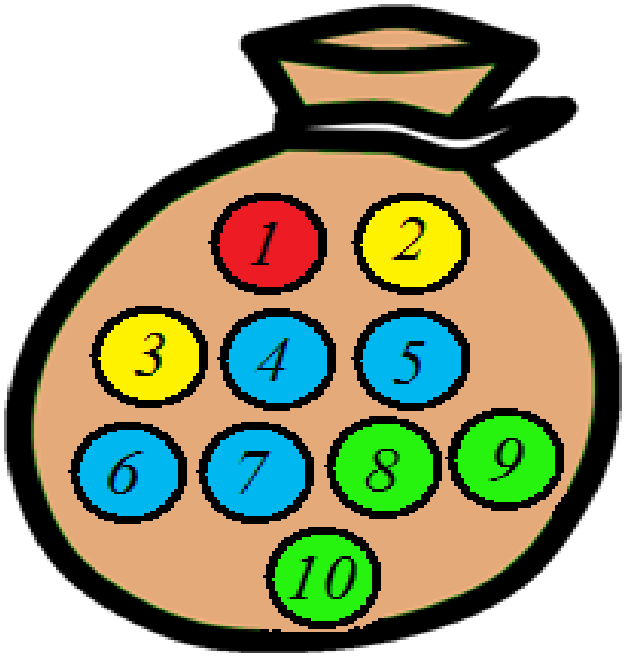
Experiment

Draw 2 balls from the bag on the right one by one with replacement

Ex 7:

Experiment

Draw 2 balls from the bag on the right one by one without replacement



Ex 8: At Greg's sandwich shop you build sandwiches by choosing a bread, a deli meat, and a type of cheese. (you must choose one of each and ONLY one of each). Here are the available choices for each selection.
Breads: White, Sourdough
Deli Meats: Chicken, Turkey, Roast Beef
Cheeses: American, Cheddar, Provolone, Swiss

a) List a few sandwiches that can be made at Greg's shop.
b) How many different sandwiches can be made at Greg's shop?
c) How many different sandwiches can be made at Greg's shop that have chicken in them?
d) If a sandwich is selected from Greg's shop at random, what is the probability that it has chicken in it?

Ex 9: License Plates

a) How many 7 character license plates can be made where the 2^{nd}, 3^{rd}, and 4^{th} characters are letters and the rest are numbers?

b) How many 7 character license plates can be made where the 2^{nd}, 3^{rd}, and 4^{th} characters are letters, the rest are numbers, and no repetition is allowed?

c) How many 7 character license plates can be made where the 2^{nd}, 3^{rd}, and 4^{th} characters are letters, the rest are numbers, and the letters can repeat but the numbers cannot?

d) What is the probability that a randomly selected 7 character license plate where the 2^{nd}, 3^{rd}, and 4^{th} characters are letters and the rest are numbers has no repetition in its characters?

e) What is the probability that a randomly selected 7 character license plate where the 2^{nd}, 3^{rd}, and 4^{th} characters are letters and the rest are numbers has no repetition in its numbers?

Ex (to motivate the factorial, permutation, & combination rules):

1) Three books (titled Algebra, Calculus, and Statistics) are to be put in a row on a shelf. How many different arrangements are possible?

2) You have a shelf that only has room for 3 books, but you have 5 books to choose from (titled Algebra, Calculus, Geometry, Trigonometry, and Statistics). How many ways are there to arrange 3 of these books in a row on the shelf?

3) Out of a group of 5 people (named Albert, Candice, George, Tom, and Sam) you are going to choose 3 people to be on a committee. How many different committees can be chosen?

The Factorial Rule

The number of ways to arrange *n* objects in a row is $n!$

(all objects are used, order matters, no repetition allowed)

The Permutation Rule

The number of ways to select *r* objects out of *n* total and arrange them in a row is ${}_nP_r$

(*r* objects are used out of *n* total, order matters, no repetition allowed)

The Combination Rule

The number of ways to select *r* objects out of *n* total and arrange them in a row is not counting order is ${}_nC_r$

(*r* objects are used out of *n* total, order doesn't matters, no repetition allowed)

Counting Examples (Mixed)

Ex 9: How many 5 card poker hands are there?

Ex 10: Out of a group of 9 people, 5 people are going to be selected and lined up in a row to take a picture. How many different pictured are possible?

Ex 11: All people in a class of 9 people are going to line up in a row to take a picture. How many pictures are possible?

More Counting Examples (Mixed)

Ex 12: There are 12 horses in a horse race. A trifecta is a bet where you have to pick the horse who will come in 1^{st}, 2^{nd}, and 3^{rd} in the correct order otherwise you lose. How many trifecta bets are possible? If each trifecta bet has the same chance of being a winning bet and you bet 18 trifecta bets, what is the probability that you will win one of your bets?

Ex 13: Out of a group of 10 people (the population), 5 people are going to be selected for a sample. How many different samples can be drawn?

Ex 14: In a 6-horse horse race, how many different outcomes of the race are possible (assuming you care about which place each horse came in)